# C3 Paper E – Marking Guide

**1.** *(i)* $0.17 < x < 0.23$ — M1 A1

*(ii)* $0.17 < 0.95^n < 0.23$, $\ln 0.17 < n \ln 0.95 < \ln 0.23$ — M1

$\frac{\ln 0.17}{\ln 0.95} > n > \frac{\ln 0.23}{\ln 0.95}$, $28.65 < n < 34.55$ — M1

$\therefore\ n = 29, 30, 31, 32, 33, 34$ — A1 **(5)**

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**2.** $= \pi\int_0^2 x^2(2-x)\ dx = \pi\int_0^2 (2x^2 - x^3)\ dx$ — M1

$= \pi[\frac{2}{3}x^3 - \frac{1}{4}x^4]_0^2$ — M1 A1

$= \pi[(\frac{16}{3} - 4) - (0)] = \frac{4}{3}\pi$ — M1 A1 **(5)**

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**3.** $2(\text{cosec}^2 y - 1) + 5\ \text{cosec}\ y + \text{cosec}^2 y = 0$ — M1

$3\ \text{cosec}^2 y + 5\ \text{cosec}\ y - 2 = 0$

$(3\ \text{cosec}\ y - 1)(\text{cosec}\ y + 2) = 0$ — M1

$\text{cosec}\ y = -2$ or $\frac{1}{3}$ (no solutions) — A1

$\sin y = -\frac{1}{2}$ — M1

$y = 180 + 30, 360 - 30$

$y = 210, 330$ — A2 **(6)**

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**4.** *(i)* $\frac{dx}{dy} = 1 \times \sqrt{1-2y} + y \times \frac{1}{2}(1-2y)^{-\frac{1}{2}} \times (-2)$ — M1 A1

$= \sqrt{1-2y} - \frac{y}{\sqrt{1-2y}} = \frac{(1-2y)-y}{\sqrt{1-2y}} = \frac{1-3y}{\sqrt{1-2y}}$ — M1

$\frac{dy}{dx} = 1 \div \frac{dx}{dy} = \frac{\sqrt{1-2y}}{1-3y}$ — A1

*(ii)* $y = -1,\ x = -\sqrt{3}$, grad $= \frac{1}{4}\sqrt{3}$ — B1

$\therefore\ y + 1 = \frac{1}{4}\sqrt{3}\,(x + \sqrt{3})$ — M1

$4y + 4 = \sqrt{3}\,x + 3$

$\sqrt{3}\,x - 4y - 1 = 0$ $\quad [p = -4, q = -1]$ — A1 **(7)**

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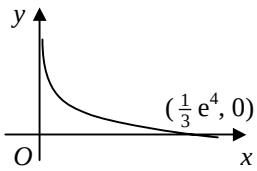
**5.** *(i)* $4 - \ln 3x = 0$

$\ln 3x = 4$

$x = \frac{1}{3}e^4$ — M1 A1

*(ii)*




B2

*(iii)* $fg(x) = 4 - \ln 3e^{2-x} = 4 - (\ln 3 + \ln e^{2-x})$ — M1

$= 4 - \ln 3 - (2 - x)$ — M1

$= x + 2 - \ln 3$ $\quad [a = 2, b = 3]$ — A1 **(7)**

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**6.** *(i)* $= [-\frac{1}{2}e^{1-2x}]_{-1}^{0}$ — M1 A1

$= -\frac{1}{2}(e - e^3) = \frac{1}{2}e(e^2 - 1)$ — M1 A1

*(ii)* $= \int_2^4 (3x - \frac{2}{x})\ dx = [\frac{3}{2}x^2 - 2\ln|x|]_2^4$ — M1 A1

$= (24 - 2\ln 4) - (6 - 2\ln 2) = 18 - 2\ln 2$ — M1 A1 **(8)**

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**7.** *(i)* $a = 2,\ \cos x + 3\sin x = b\cos x\cos c + b\sin x\sin c$
$b\cos c = 1,\ b\sin c = 3$ M1
$\therefore\ b = \sqrt{1^2 + 3^2} = \sqrt{10}$ A1
$\tan c = 3,\ c = 1.25$ (3sf) A1
$\therefore\ \mathrm{f}(x) = 2 + \sqrt{10}\cos(x - 1.25)$

*(ii)* $2 + \sqrt{10}\cos(x - 1.249) = 0, \quad \cos(x - 1.249) = -\frac{2}{\sqrt{10}}$ M1
$x - 1.249 = \pi - 0.8861,\ \pi + 0.8861 = 2.256,\ 4.028$ M1
$x = 3.50,\ 5.28$ (3sf) A2

*(iii)*

| $x$ | 0 | 0.5 | 1 | 1.5 | 2 |
|---|---|---|---|---|---|
| $\mathrm{f}(x)$ | 3 | 4.3159 | 5.0647 | 5.0632 | 4.3117 |

M1
$I \approx \frac{1}{3} \times 0.5 \times [3 + 4.3117 + 4(4.3159 + 5.0632) + 2(5.0647)]$ M1
$= 9.16$ (3sf) A1 **(10)**

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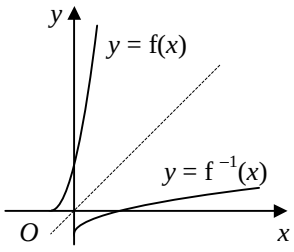
**8.** *(i)* $= 2[x^2 + 2x] + 2 = 2[(x + 1)^2 - 1] + 2$ M1
$= 2(x + 1)^2$ A1

*(ii)* translation by 1 unit in negative $x$ direction
stretch by scale factor of 2 in $y$ direction (either first) B3

*(iii)* $y = 2(x + 1)^2, \quad \frac{y}{2} = (x + 1)^2$
$x + 1 = \pm\sqrt{\frac{y}{2}}, \quad x = -1 \pm \sqrt{\frac{y}{2}}$ M1
$\therefore\ \mathrm{f}^{-1}(x) = -1 + \sqrt{\frac{x}{2}},\ x \in \mathbb{R},\ x \geq 0$ A2

*(iv)*




B2

$y = \mathrm{f}^{-1}(x)$ is reflection of
$y = \mathrm{f}(x)$ in line $y = x$ B1 **(11)**

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**9.** *(i)* $t = 10,\ T = 18 \quad \Rightarrow \quad 18 = 5 + Ae^{-10k}$ (1)
$t = 60,\ T = 12 \quad \Rightarrow \quad 12 = 5 + Ae^{-60k}$ (2) M1
(1) $\Rightarrow \quad A = \frac{13}{e^{-10k}} = 13e^{10k}$ M1
sub (2) $\Rightarrow \quad 7 = 13e^{10k} \times e^{-60k}$
$e^{-50k} = \frac{7}{13}$ A1
$\therefore\ k = -\frac{1}{50}\ln\frac{7}{13} = 0.0124$ (3sf) M1 A1
$\therefore\ A = 13e^{10 \times 0.01238} = 14.7$ (3sf) A1

*(ii)* $T = 5 + 14.71e^{-0.01238t}$
$\frac{dT}{dt} = -0.01238 \times 14.71\, e^{-0.01238t} = -0.1822e^{-0.01238t}$ M1 A1
when $t = 20, \quad \frac{dT}{dt} = -0.1822e^{-0.01238 \times 20} = -0.142$ M1
$\therefore$ temperature decreasing at rate of 0.142 °C per minute (3sf) A1

*(iii)* $T = 5 + 14.71e^{-0.01238(t - 60)}$ M1
$= 5 + 14.71e^{0.7428 - 0.01238t}$
$= 5 + 14.71e^{0.7428} \times e^{-0.01238t}$ M1
$= 5 + 30.9e^{-0.01238t}, \quad B = 30.9$ (3sf) A1 **(13)**

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Total **(72)**

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